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1983 J. Phys. A: Math. Gen. 16 2881

(http://iopscience.iop.org/0305-4470/16/12/034)

J. Phys. A: Math. Gen. 00 (1981) 2881–2882. Printed in Great Britain

COMMENT

On the location of the critical point of the q-state Potts models on the decorated hypercubic lattice

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Received 11 February 1983, in final form 15 April 1983

Abstract. Recently a critical condition for the q-state Potts ferromagnet on the hypercubic lattice was obtained. We extend this result to both ferromagnetic and antiferromagnetic Potts models on the bond-decorated hypercubic lattice.

The model considered in this comment is the q-state Potts model described by the Hamiltonian

$$\mathscr{H} = -\varepsilon \sum_{\langle ij \rangle} \delta_{kr}(\sigma_i, \sigma_j). \tag{1}$$

Here $\sigma_i = 1, 2, \ldots, q$ specifies the spin state at the ith site, $\delta_{kr}(\alpha, \beta)$ is the Kronecker delta and the sum is taken over the nearest-neighbour sites on the lattice. The Potts model is ferromagnetic if $\varepsilon > 0$ and antiferromagnetic if $\varepsilon < 0$. Recently it was shown (Hajduković 1983) that the critical coupling $K_c \equiv \varepsilon/kT_c$ (k being the Boltzmann constant and T_c the critical temperature) of a q-state Potts ferromagnet on the hypercubic lattice in d dimensions may be considered as a solution of

$$e^{dK} - 2^{1/(d-1)} e^{dK/2} = q - 1. \tag{2}$$

The critical condition (2) is known to be exact for $d = 2$ dimensions, and it is at least an excellent approximation for all $q \geqslant 2$ in $d \geqslant 3$ dimensions (including the case when q is a continuous variable). In the present paper we extend the critical condition (2) to both ferromagnetic and antiferromagnetic Potts models on the bond-decorated hypercubic lattice. The antiferromagnetic model has attracted increasing attention after a rescaling prediction by Berker and Kadanoff (1980) that a distinctive low-temperature phase in which correlations decay algebraically with distance can exist in these systems. For the q-state antiferromagnetic Potts model this behaviour is permitted either when the spatial dimensionality d is sufficiently high, or for a fixed d, when q is less than a cut-off value $q_0(d)$. However this cut-off value and the critical point are not known for $d \geqslant 3$.

Let us consider a bond-decorated hypercubic lattice in d dimensions with pure antiferromagnetic ($K^* < 0$) or pure ferromagnetic interactions ($K^* > 0$). Taking the partial traces over the bond-decorating sites leads to an effective hypercubic lattice. This hypercubic lattice has ferromagnetic interactions K determined by (see for example Wu 1982)

$$e^{K} = (e^{2K^*} + q - 1)/(2e^{K^*} + q - 2) \tag{3}$$

which gives

$$e^{K^*} = e^K \mp [(e^K - 1)(e^K + q - 1)]^{1/2}. \tag{4}$$

If we introduce the critical value e^{K_c} determined by (2), equation (4) gives the critical point $e^{K_c^*}$ of the Potts model on the decorated hypercubic lattice. The minus (plus) sign in (4) corresponds to the antiferromagnetic (ferromagnetic) model.

The cut-off value $q_0(d)$ for the antiferromagnetic model follows immediately from (3) by setting $e^{K^*} = 0$ and using (2). It is a solution of

$$[(q-1)/(q-2)]^d - 2^{1/(d-1)}[(q-1)/(q-2)]^{d/2} = q - 1. \tag{5}$$

The cut-off values for $d = 2, 3, 4$ are

$$q_0(2) = \tfrac{1}{2}(3+\sqrt{5}) = 2.618\ldots \qquad q_0(3) = 3.282 \qquad q_0(4) = 3.801.$$

The cut-off value $q_0(2)$ is the same as the exact result (Wu 1980). Let us note that the cut-off value $q_0'(2) = 3$ of the square lattice (Baxter 1982) is higher than that of the decorated square lattice. We expect this to be true in the general case, and if it is so the cut-off value of the decorated hypercubic lattice may be considered as a lower bound for the cut-off value of the corresponding hypercubic lattice. In particular, for the simple cubic lattice it indicates that an ordered low-temperature phase must exist for $q = 3$. This is in agreement with the Monte Carlo simulation (Banavar *et al* 1980).

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